An Equivalent Circuit Model for the Coplanar Waveguide Step Discontinuity

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Abstract

An approximate analytical model for the CPW Step Discontinuity is presented which includes a lumped shunt capacitance as the dominant effect. However, the magnitude is much lower than previously published work. Good agreement is obtained with data from extensive 3-D field simulations, and verified by a limited number of measurements on alumina.

I. Introduction & Background

The Coplanar Waveguide (CPW) Step Discontinuity has been analysed numerically by Kuo & Itoh [1] and Chen & Gao [2], giving S-parameters for a single structure only. Itoh predicted $|S_{11}| \approx 0.25$ in the frequency range 26 to 40 GHz for $\varepsilon_r = 10$, whilst Chen predicts values of $|S_{11}| \approx 0.3$ at X-band, also for $\varepsilon_r = 10$. Assuming only a shunt capacitative discontinuity, these correspond to approximately 0.07 pF and 0.2 pF respectively.

Experimental results have been published by Simons & Ponchak [3]. They measured a range of steps on duroid ($\varepsilon_r = 2.2$) and show capacitances from 0.02 to 0.08 pF. Scaling to alumina, this would become approximately 0.07 to 0.27 pF.

Finally, Jansen [4] has indicated from a simulation using UNISIM™ that the shunt capacitance is negligible and the dominant discontinuity is a series inductance.

These results are compared in Fig. 1, showing the wide variation of the predictions, and hence the need to conduct an investigation.

This paper describes a comprehensive modelling of the CPW Step Discontinuity for a large range of physical parameters (dielectric constant, step ratio, etc), with data from three independent sources:

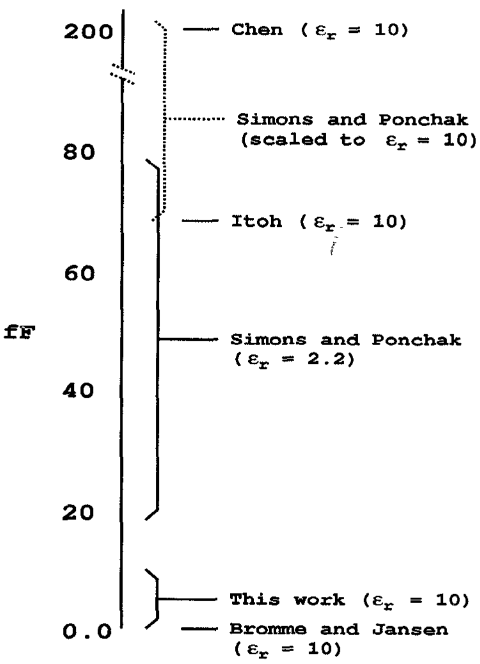

Fig 1 - Capacitance of Step Discontinuity: Comparison of Previous Works

1. An analytical technique based on a conformal mapping and a parallel plate waveguide approximation. The authors are not aware of any previous attempt to derive analytic expressions for the CPW Step Discontinuity.

2. 3-D electromagnetic field simulations.

3. Measurements of a limited number of step structures on alumina, both individually and in a filter type arrangement.

The analytical technique results in relatively simple equations, which can be easily programmed and enable values of capacitance to be rapidly calculated for a particular physical structure.

II. Analytical Technique

A symmetric step change in width of the centre conductor of grounded CPW [Fig. 2a] is considered. The coplanar lines leading to the step are approximated by two parallel plate waveguides [Fig. 2b] with the distance, w, between the plates set to be the same as the gap between the inner and outer conductors of the coplanar structure. The widths of each parallel plate guide, x, are such that the capacitance per unit length (and hence the propagation characteristics) are the same as the corresponding coplanar lines. The capacitance per unit length of each coplanar line is found using the conformal mapping due to Ghioni and Naldi [5]:

$$c(k) = 2\varepsilon_0 \varepsilon_{eff}\left(\frac{K}{K'}(k_3) + \frac{K}{K'}(k)\right) \qquad \text{[Eqn. 1]}$$

where $k = \dfrac{s}{s + 2w}$ and $k_3 = \dfrac{\tanh\left(\dfrac{\pi k y}{4h}\right)}{\tanh\left(\dfrac{\pi y}{4h}\right)}$

with $\varepsilon_{eff} = 1 + (\varepsilon_r - 1) \cdot \dfrac{\frac{K}{K'}(k_3)}{\frac{K}{K'}(k_3) + \frac{K}{K'}(k)}$

and $\frac{K}{K'}(k)$ is the elliptic integral ratio.

Therefore, for capacitance per unit length equivalence:

$$\frac{\varepsilon_0 x_1}{w_1} = c(k_1) \quad \text{and} \quad \frac{\varepsilon_0 x_2}{w_2} = c(k_2) \qquad \text{[Eqns. 2,3]}$$

Another conformal mapping result [6], for the discontinuity capacitance per unit width of a step in height from w_1 to w_2 of a parallel plate guide [Fig. 2c] is then used - this is a reasonable approximation to the CPW step as in the CPW the majority of the field is between the inner and outer conductors with some fringing.

$$c_{//step}(\alpha) = \frac{\varepsilon_0}{\pi} \cdot \left[\left(\frac{\alpha^2+1}{\alpha}\right) . \ln\left(\frac{1+\alpha}{1-\alpha}\right) - 2 . \ln\left(\frac{4\alpha}{1-\alpha^2}\right)\right]$$

where $\alpha = \dfrac{w_2}{w_1}$, $\alpha < 1$. [Eqn. 4]

The difficulty arises when the equivalent width x of the parallel plate guide is calculated - it is not the same on both sides of the step, so initially the average of x_1 and x_2 was used to find the actual CPW step capacitance,

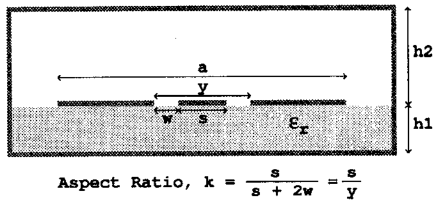

Fig 2a - Coplanar Waveguide

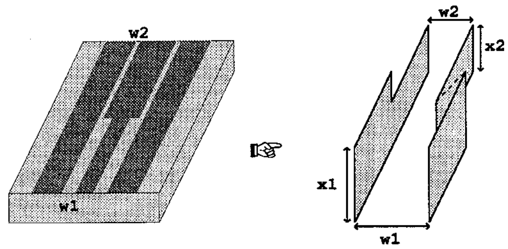

Fig 2b - Plane Waveguide Approximation of Step Discontinuity

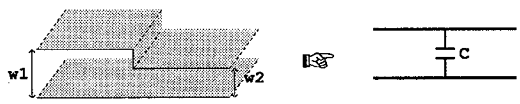

Fig 2c - Plane Waveguide Step and Equivalent Circuit

although there is no particular justification for doing this.

$$\bar{x} = \frac{x_1 + x_2}{2} \qquad \text{[Eqn. 5]}$$

$$C_{CPWstep} = \bar{x} . C_{//step}\left(\frac{w_2}{w_1}\right) \qquad \text{[Eqn. 6]}$$

Fig. 3 shows the capacitance $C_{CPWstep}$ as a function of the step ratio k_2 / k_1, for $\varepsilon_r = 2.2$ and $\varepsilon_r = 10$. These have k_2 fixed at 0.9 and k_1 varied from 0.1 to 0.8, giving a step ratio from 1 to 9. Also shown are the numerically derived results from the 3-D field simulations [§ III], from which it can be seen that there is reasonable agreement but the shape of the curves needs softening and other corrections.

This was achieved by replacing the average of the equivalent widths x_1 and x_2 [Eqn. 5] with an expression that uses the weighted average of the gaps w_1 and w_2, and determining an 'average' width $\bar{x}$ from that. The weighting used includes several intuitive adjustments and second order terms involving k_1 and k_2:

$$\bar{x} = fn(k_1, k_2, \ldots) \qquad \text{[Eqn. 7]}$$

To the authors' knowledge, this is the first published expression for the CPW Step Discontinuity capacitance in terms of the physical parameters. The curves are, however, the same shape as those from Simons and Ponchak [3], but of significantly lower magnitude. For typical steps on alumina, the capacitance is of the order of 10 fF.

No allowance has been made for the effects of metallisation thickness, however this would only become noticeable with very small gap widths (k > 0.8).

A similar application of this method to the dual structure, Coplanar Strips (CPS), will lead to a value for the total series inductance of the CPW Step, as suggested by Wolff [7].

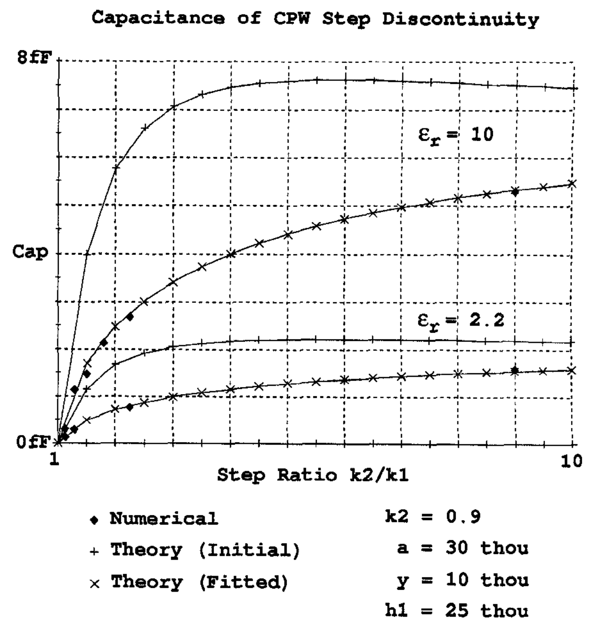

Fig 3 - Numerical and Theoretical Results

The empirically adjusted model [Eqn. 7] has been made to agree well with this simulation data over a wide range of physical parameters; typical adjusted model curves are also shown in Fig. 3.

III. Numerical Simulations

Extensive 3-D electromagnetic field simulations have been carried out using the *em*™ simulator. Using a fairly fine subsection size for good accuracy, it was discovered early on that the discontinuity capacitance was almost independent (ie < 1%) of frequency over the range 1 to 40 GHz, so simulations were then restricted to 10 GHz to save time. This agrees with observations from Wolff [8] that CPW discontinuities are not significantly dispersive. The subsection size was made progressively smaller, until the results no longer changed significantly. Using this accuracy, a carefully selected range of steps was chosen and simulated.

The effects of varying the independent parameters ε_r, a, h_1, k_1 and k_2 upon capacitance have been investigated, with the following conclusions:

ε_r The variation with dielectric constant is found to be linear over the range 2 to 14 as expected, ie $C \propto (\varepsilon_r+1) / 2$.

a The outer ground plane width has no effect from $a / y = 2.5$ to $a / y = 6$.

h_1 The dielectric thickness has little effect on the capacitance for values $h_1 / y > 1$.

k_2 / k_1 There are actually two variables to consider, as both k_2 / k_1 and k_2 can vary. The step ratio directly affects the capacitance. As k_2 / k_1 tends to infinity, the capacitance tends to the open circuit capacitance for a line of aspect ratio k_2.

IV. Experimental Work

A number of CPW step structures were fabricated on alumina, together with through lines for de-embedding purposes. Using ground-signal-ground wafer probes, previously calibrated using the manufacturer's calibration substrate, the CPW structures were measured directly. The discontinuity was then de-embedded by removing the lengths of feed strips leading to the step. The parameters of these feed strips were calculated using the method presented by Shih [9], after measuring two through lines which were of the same dimensions.

The results indicate that the capacitance is less than 20 fF (exact values could not be deduced by this technique due to the limitations of measurement accuracy). However, the measurements do confirm that previous calculations of the capacitance have led to too high a value since these values of capacitance would easily have been measurable using this technique.

Another test substrate has been fabricated containing periodic stepped impedance filter structures. By measuring the resonant frequencies (which can be done very accurately), the equivalent circuit values of the discontinuity can be found, using a procedure similar to that presented by Rizzoli [10], and these results will be presented at the Symposium.

V. Conclusions

A simple model for the shunt capacitance of a CPW Step Discontinuity has been developed, using an approximate analytical technique, and adjusted to fit data from extensive 3-D field simulations. Good agreement is obtained with a limited number of measurements.

The magnitude of the discontinuity is found to be smaller than several previous works have predicted, but it is nethertheless of the same order as a microstrip step in a comparable situation. These effects need to be taken into account in any serious design, and the model presented, when incorporated into CAD circuit analysis programs, will enable rapid evaluation of performance.

Acknowledgements

This work has been carried out with the support of the Procurement Executive of the Ministry of Defence, UK.

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The authors wish to thank Dr John Walker for his helpful advice during the preparation of this paper.

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